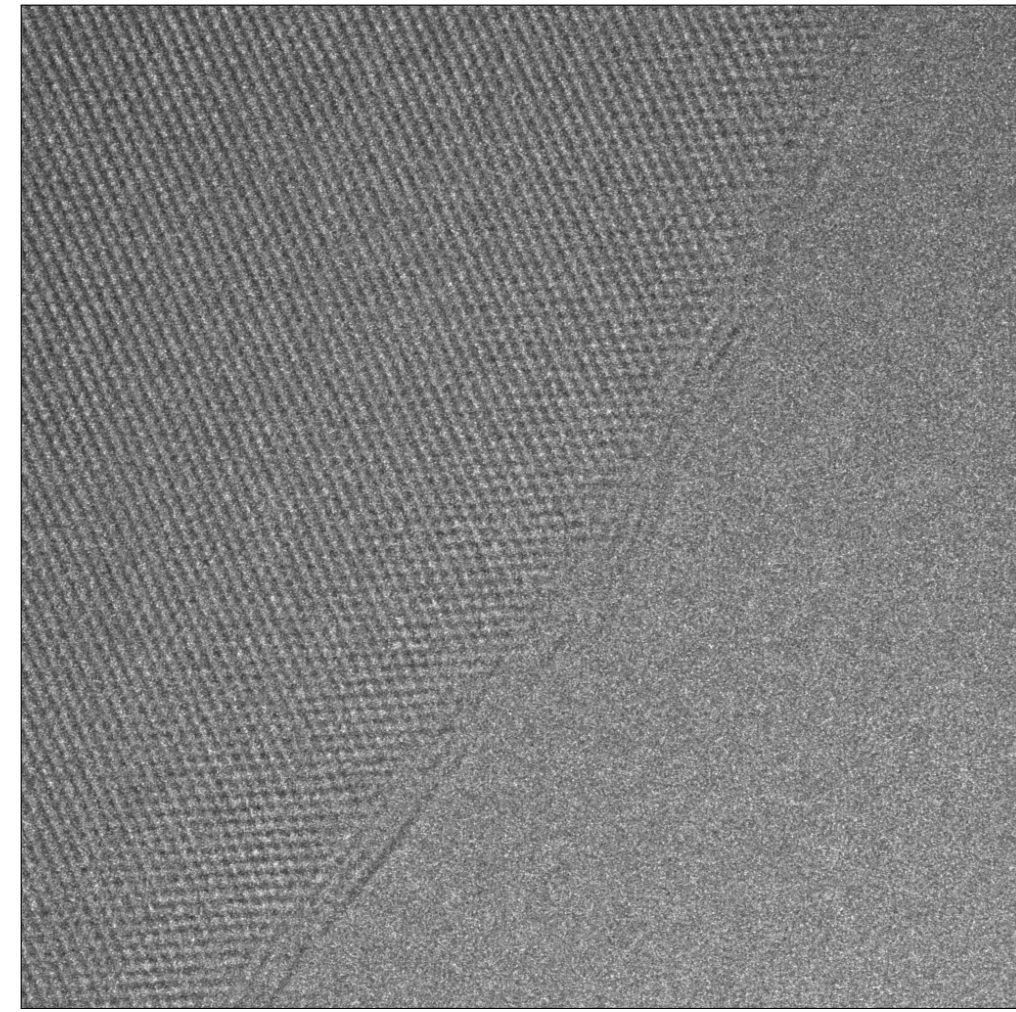


# Parallel Prefix Algorithms for the Registration of Arbitrarily Long Electron Micrograph Series

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## Motivation



The process of aluminum oxidization, acquired with a transmission electron microscope. Courtesy of the University of Manchester.

- ▶ The interaction of electrons with the specimen is mapped to a display device
- ▶ The quality of images is limited by changes induced by electrons and movement of the sample
- ▶ Solve both problems by acquiring a series of low dose frames and processing it to extract information
- ▶ Image registration is computationally expensive - can we parallelize that process?

## Image registration

**Goal:** find rigid deformations  $\phi_{0,i}$  for frames  $f_1, \dots, f_n$

$$f_0 \approx f_i \circ \phi_{0,i}$$

**Neighboring frames:** apply registration function **A**

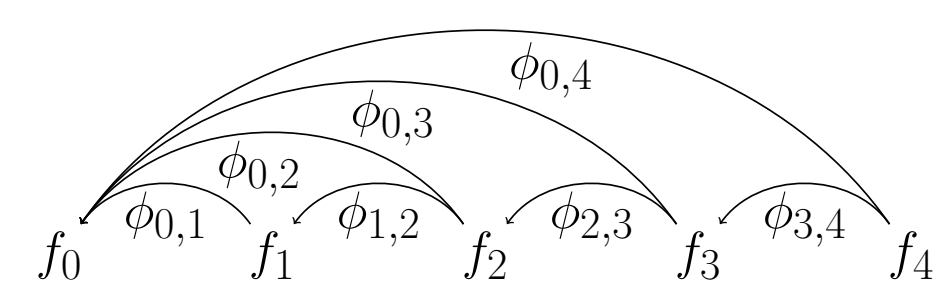
$$\phi_{i,i+1} = \mathbf{A}(f_i, f_{i+1})$$

**Non-neighboring frames:** use registration function **B**

$$\phi_{i,j} = \mathbf{B}(\phi_{i,k}, \phi_{k,j})$$

**Prefix sum registration:** with a binary operator  $\odot_B$

$$\phi_{0,i} = \phi_{0,1} \odot_B \phi_{1,2} \odot_B \dots \odot_B \phi_{i-1,i}$$



## Parallel image registration

- ▶ A **trivially parallelizable** preprocessing step on neighboring pairs of frames

$$f_0, f_1, \dots, f_n \rightarrow \phi_{0,1}, \phi_{1,2}, \dots, \phi_{n-1,n}$$

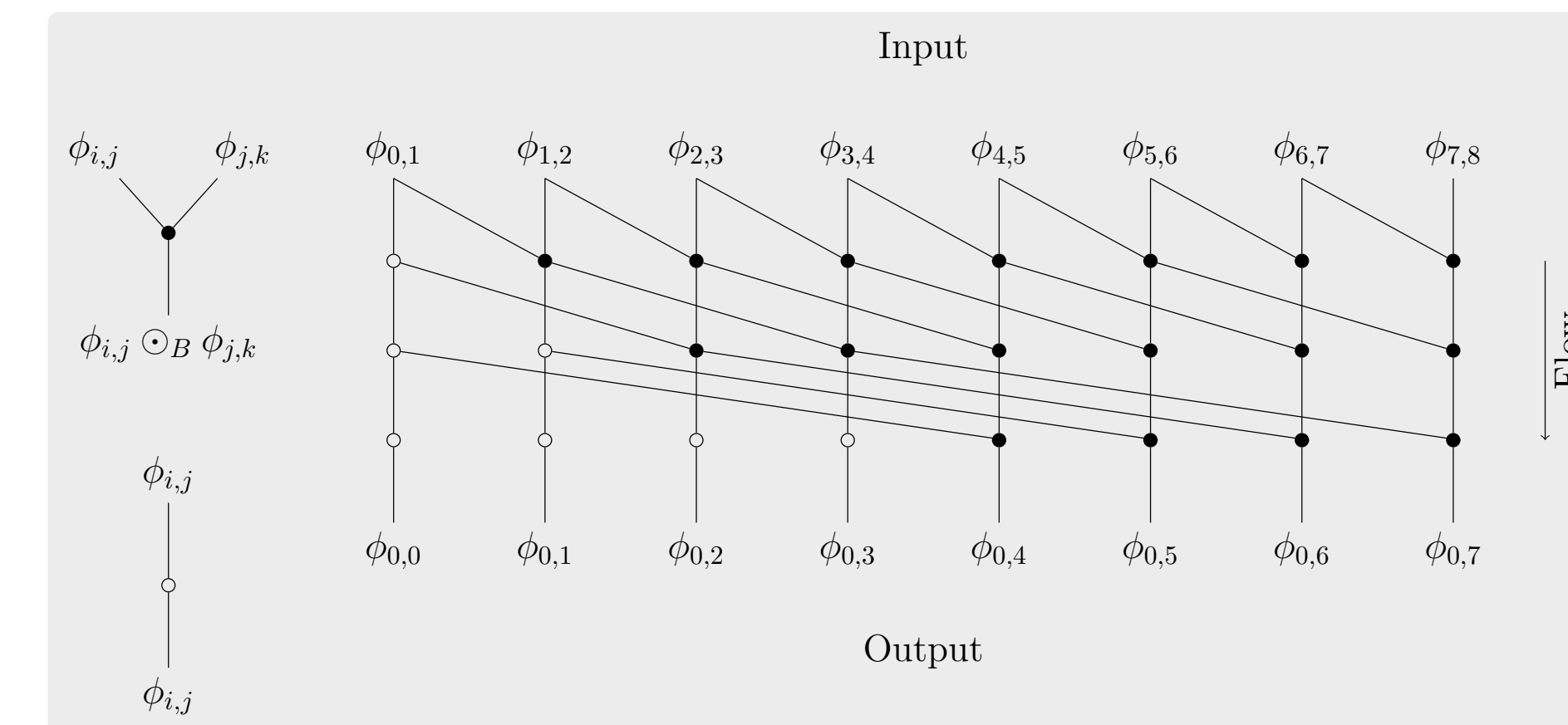
- ▶ A prefix sum registers each frame to the first image, producing a new sequence

$$\phi_{0,1}, \phi_{0,2}, \dots, \phi_{0,n}$$

**a parallel prefix sum is necessary**

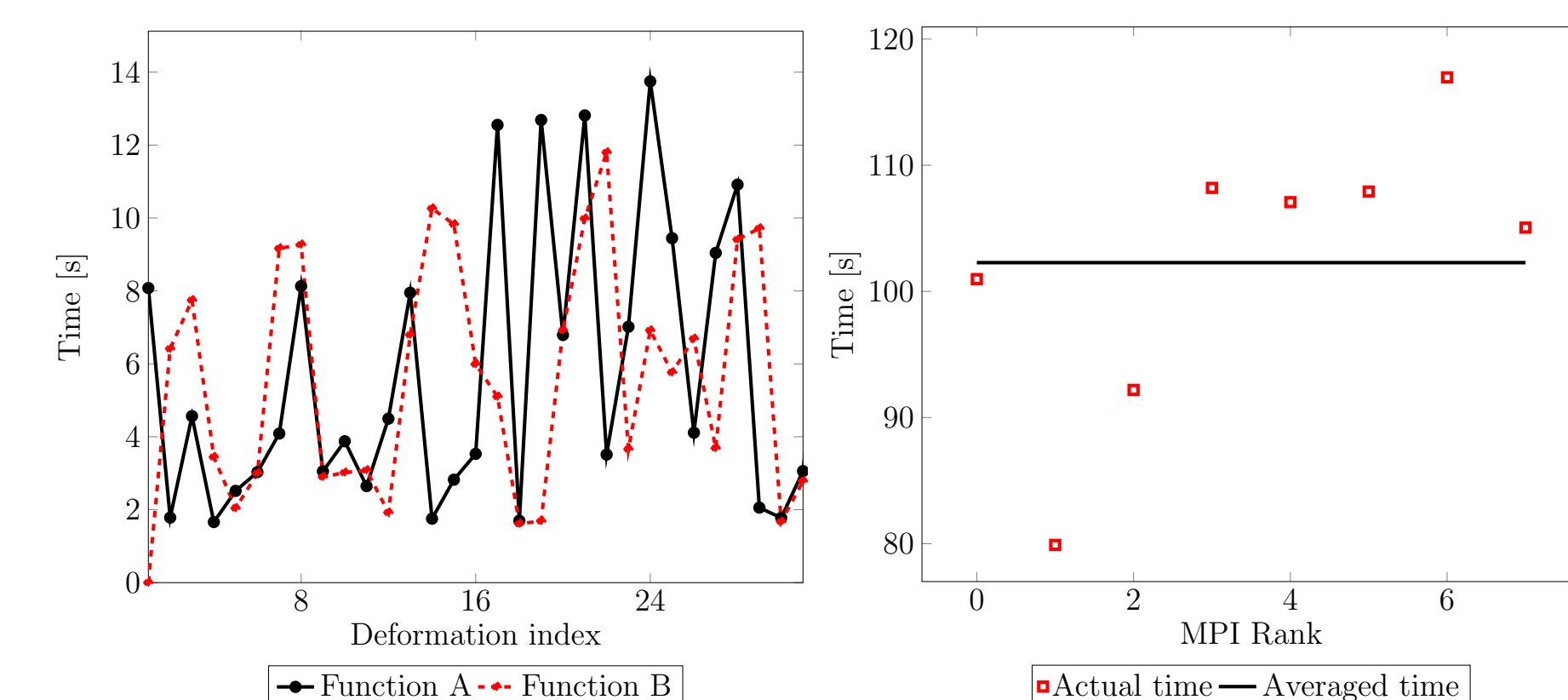
## Parallel prefix sum

- ▶ Multiple algorithms for a parallel prefix sum exist
- ▶ A decrease in span is achieved by performing more work
- ▶ Scan algorithms with  $\log_2 N$  span are optimal, but they can not be work-efficient



Kogge–Stone(Hills–Steel) parallel prefix sum  
 $S(N) = \log_2 N$

## Image registration operator



(1) Registration times on 64 images (2) Function B on 128 deformations on 8 MPI ranks

Existing parallel prefix sum algorithms

- ▶ Are simple and not computationally intensive
- ▶ Have a stable and deterministic execution time

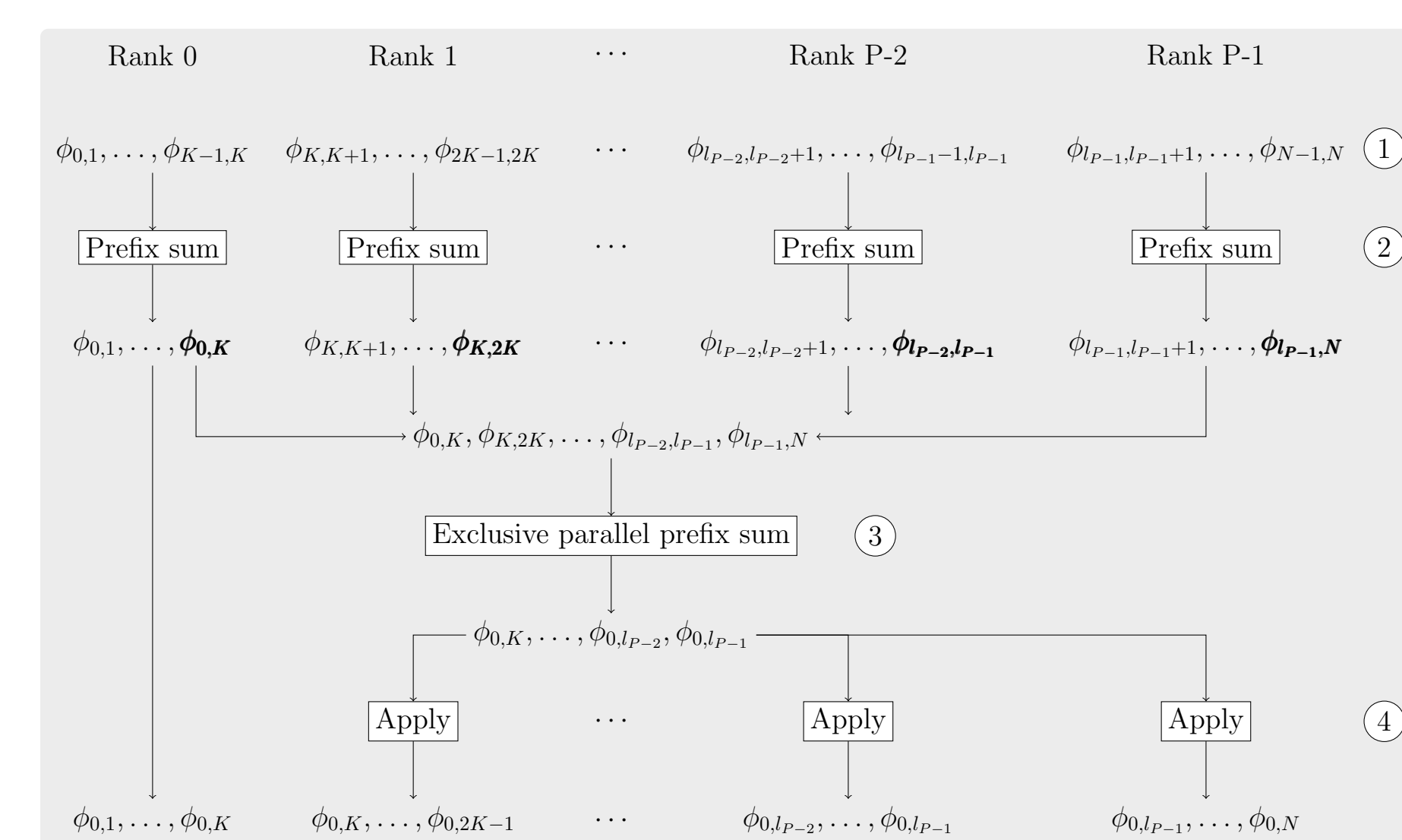
In our problem, the image registration operator

- ▶ Is complex and computationally intensive (1)
- ▶ Has an unpredictable and unbalanced execution time (2)

## Parallel prefix sum for image registration

- ▶ Communication cost  $\ll$  computation cost  $\Rightarrow$  do not optimize for communication
- ▶ Reduce computation time  $\Rightarrow$  minimize span
- ▶ Variations in execution time are not known a priori  $\Rightarrow$  distribute data equally between MPI ranks

## Distributed prefix sum



- Rank  $J$  obtains a sequence of data  $\phi_{J,J+1}, \phi_{J+1,J+2}, \dots, \phi_{J+1-1,J+1}$  with  $K = \frac{N}{P}$  elements, where  $J = J \cdot K$  is a left index for rank  $J$
- Rank  $J$  performs a local prefix sum, reducing sequence to  $\phi_{J,J+1}$

$$S(N, P) = \frac{N}{P} - 1$$

- A global exclusive prefix sum on reduced values  $\phi_{J,J+1}$  is performed, rank  $J$  obtains a result  $\phi_{0,J}$

$$S(N, P) = C_1 \log_2 P$$

- The global result  $\phi_{0,J}$  and local values  $\phi_{J,J+i}$  are combined to form final results  $\phi_{0,J+i}$

$$S(N, P) = \frac{N}{P}$$

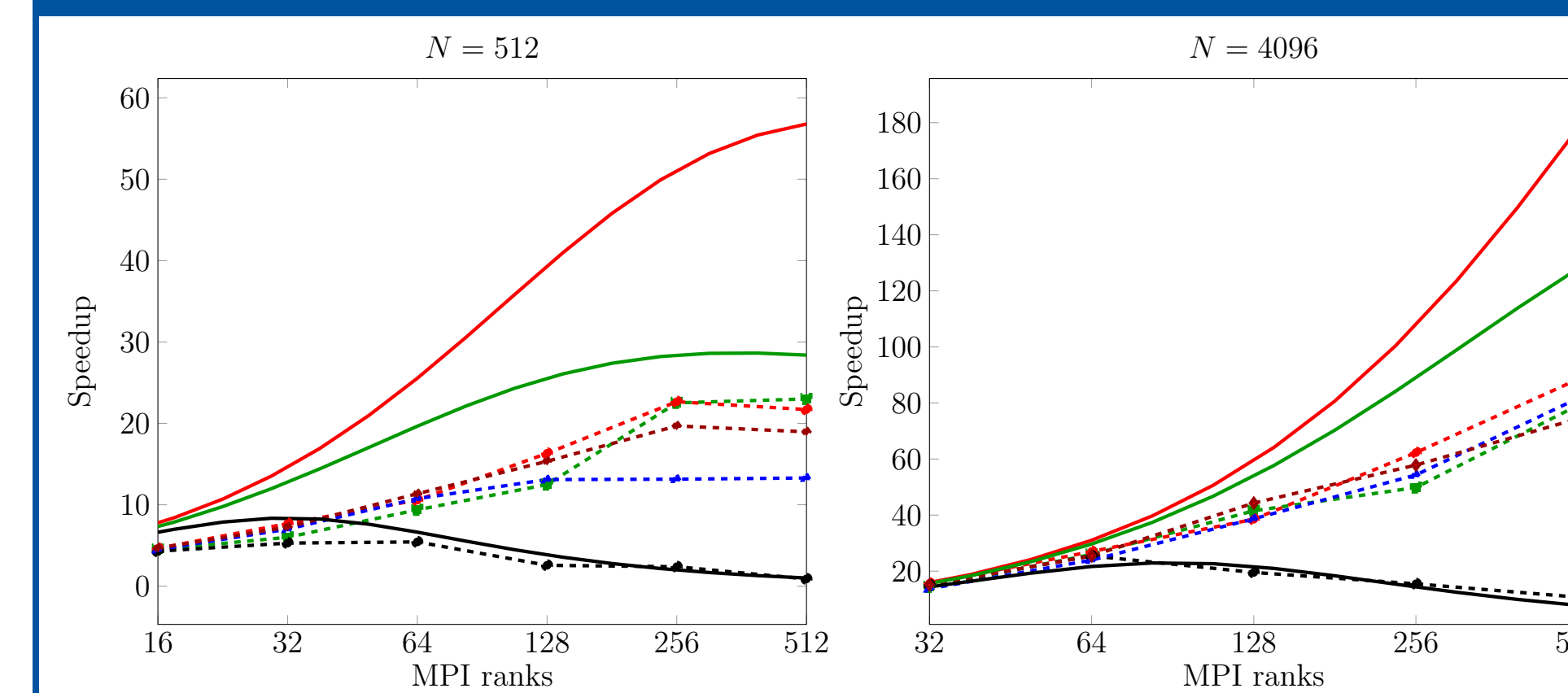
The span of a prefix sum for  $N$  images on  $P$  ranks

$$S(N, P) = \frac{N}{P} + C_1 \log_2 P + \frac{N}{P} + C_2$$

The attainable speedup is bounded from above by

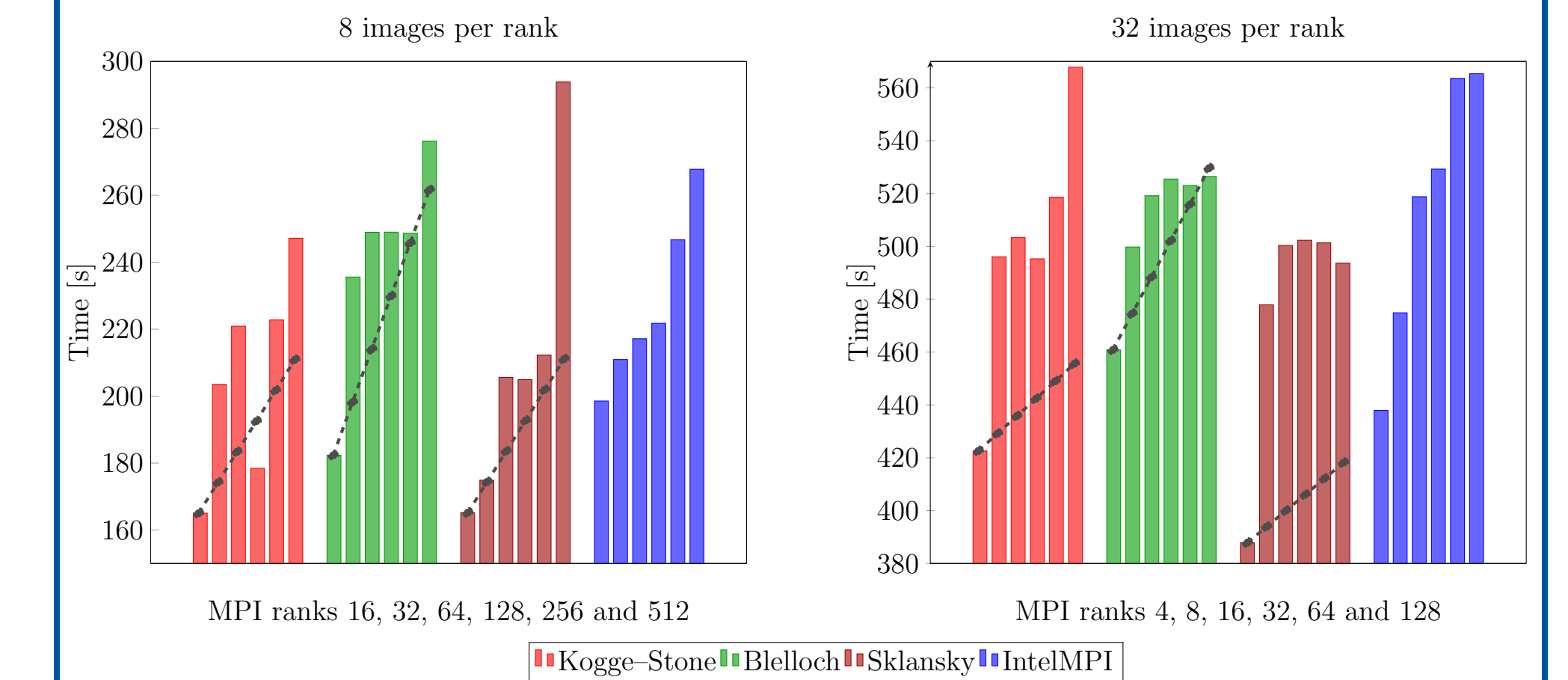
$$SP(N, P) = \frac{S(N, 1)}{S(N, P)} = \frac{N - 1}{\frac{N}{P} + C_1 \log_2 P + C_2}$$

## Strong scaling



Theoretical and measured speedups are represented with solid and dashed lines, respectively.

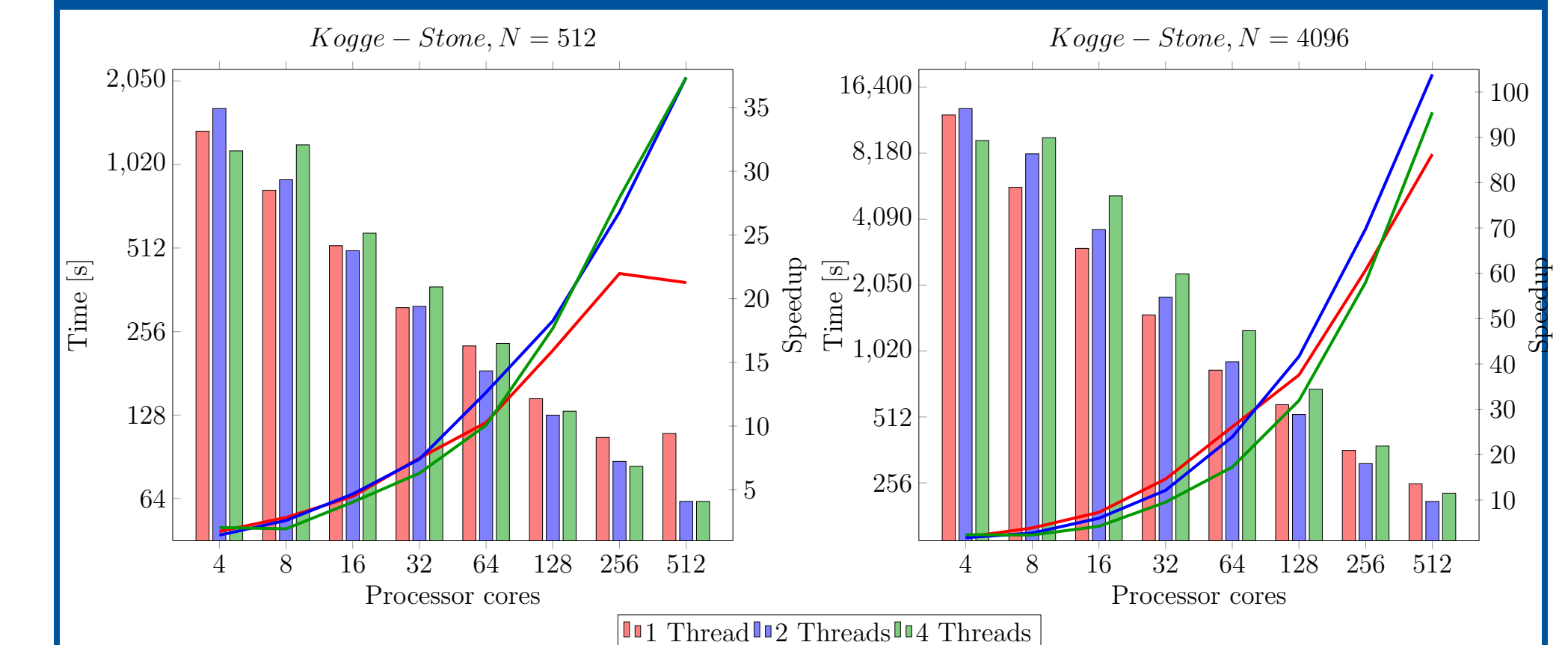
## Weak scaling



Linear weak scaling is not attainable, since

$$S(k \cdot N, k \cdot P) = S(N, P) + \log_2 k \cdot C_1$$

## Multithreading



Allocate  $\frac{P}{T}$  ranks with  $T$  threads per each MPI process.

## Summary

- ▶ The benchmarking has been performed on Intel Xeon E5-2680 v2 IvyBridge processors with GCC 5.3
- ▶ Parallelization is limited by the nature of prefix sum
- ▶ Measured performance does not meet theoretical predictions because of the unbalanced workload
- ▶ Custom scan implementations outperform MPI\_Scan and MPI\_Scan from OpenMPI 1.10 and IntelMPI 2017.1
- ▶ The performance is improved by shifting resources to operator parallelization
- ▶ The image registration procedure is a novel example of a parallel prefix sum with non-trivial sum operator

## References

- ▶ Benjamin Berkels et al. (2014). "Optimized imaging using non-rigid registration". In: *Ultramicroscopy* 138, pp. 46–56
- ▶ G. E. Blelloch (Nov. 1989). "Scans As Primitive Parallel Operations". In: *IEEE Trans. Comput.* 38.11, pp. 1526–1538



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